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Observations of Tidally Coherent Diurnal and Semidiurnal Variations in the Geocenter

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Abstract

The center of mass of the Earth, about which a satellite orbits, is determined by the mass distribution of the solid Earth (including the Earth's interior), the oceans, and the atmosphere. The tracking silts, however arc all located on the lithosphere only, and so the network origin will not generally coincide with the center of mass. Changes in the vector offset between these two points, referred to as apparent geocenter motion, are due to global scale movement of mass. In this study we have extended the approach used by space geodetic techniques for determination of tidally coherent nearly diurnal and semidiurnal variations in the Earth's orientation to include geocenter variations due to ocean tides. We demonstrate the weaker observability of nearly diurnal retrograde motions in x and y, and hence the utility of their separation from the other equatorial terms. We find statistically significant terms at the 99.5% level due to O_1, P_1, K_1, and M_2. The solutions, which indicate geocenter motions at the few millimeter level per tide, compare well with predicted values from theoretical ocean tide models and from TOPEX/Poseidon radar altimetry.

Introduction

The location of the center of mass of the combined solid Earth-ocean-atmosphere system relative to the the lithosphere-based, tracking-site-defined polyhedron, generally referred to as the geocenter, undergoes variations for much the same reasons as variations in polar motion, universal time, and other geopotential terms. The causes of these variations are mass redistribution within and between the lithosphere, hydrosphere, cryosphere, and atmosphere.

Note that when we speak of variations in the geocenter position, it is in fact the relative motion of the center of mass of the whole Earth with respect to the tracking network that is measured, and in the absence of external forces, the center of mass cannot in fact accel(’rate in inertial space. Thus, in reality it is the motion of the solid Earth, on which the tracking network is located, that is II IO II I1OMYI with the techniques described in this study.

Recent work using VLBI by *Herring and Dong* [1994] and *Sovers et al.* [1993] and satellite laser ranging (SLR) by *Watkins and Eanes* [1994] have clearly demonstrated the effects of diurnal and semidiurnal tides on polar motion and universal time. The observations of the rotational response of the Earth to these global excitations provides a useful constraint on tide models. The geocenter, however, is sensitive to these same variations, and thus provides additional information distinct from that of the Earth orientation variations. In this paper, we extend the formulation of *Watkins and Eanes* [1994] to estimate nearly diurnal and nearly semidiurnal variations in the geocenter due to several significant ocean tides. The results are compared to the signal predicted by the ocean tide model of *Schwiderski* [1980] based upon tide gauge measurements and of the CSR3.0 ocean tide model of Eanes [personal communication, 1995] based 11[)011 TOPEX/Poseidon radar altimeter data. Finally, we compare to the predictions from the model of *Brosche and Wunsch* [1993] based entirely on theoretical ocean dynamics.

Method

We develop the expressions for the variations in the geocenter following closely that for Earth orientation in *Watkins and Eanes* [1994]. This involves the construction of the complex quantity g, defining the geocenter variations in the x and y coordinates, and allowing for both prograde and retrograde terms, defined as such by the tidal argument as opposed to the rotation of the Earth. The definition for variations in z is similar, but simpler, because retrograde and prograde variations are not applicable. Thus beginning with x and y:

$$g = \sum_{s=1}^{N} [g^{s+} e^{i\theta_s} + g^{s-} e^{-i\theta_s},] = x_g + iy_g$$

where θ_s is the instantaneous phase of the particular tide in question, and g^{s+} and g^{s-} are complex prograde and retrograde subdiurnal geocenter motions modulated by the tidal frequency. As above, we define the real and imaginary parts of g^{s+} and g^{s-} in terms of the usual x_g and y_g components as:

$$g^{s+} = x_g^{s+} + iy_g^{s+}$$

$$g^{s-} = x_g^{s-} + iy_g^{s-}$$

where x_g^{s+}, x_g^{s-}, y_g^{s+}, and y_g^{s-} are the real variables to be estimated using the SLR data. Similarly, high frequency variations in the z-component are modeled as:

$$z_g = \sum_{s=1}^{N}[z_g^{sc}\cos(\theta_s) + z_g^{ss}\sin(\theta_s)]$$

where z_g^{sc} and z_g^{ss} are the real parameters to be estimated. For each of the x, y, and z components, the index s varied over 5 significant nearly diurnal ocean tide frequencies K_1, S_1, P_1, O_1, and Q_1, and 4 significant nearly semidiurnal tides K_2, S_2, M_2, and N_2. These are the largest ocean tides as measured by equilibrium amplitude, and have been shown to contain the majority of power in subdiurnal Earth orientation variations. We note that the S_1 tide has a small ocean tide amplitude, but a potentially large atmospheric component, and hence we have chosen to include it.

Observability

The inseparability of retrograde nearly diurnal polar motion from nutation (or precession in the case of zero frequency) has been well documented in the discussion of VLBI observations of subdaily variations in Earth orientation. Several references have also demonstrated the inseparability of precession and nutation from the orientation of a satellite orbit in inertial space [*Lambeck*, 1971]. Hence it follows that nearly diurnal retrograde polar motion variations are inseparable from orbital variations, and hence *Watkins and Eanes* [1994] provided no solution for these terms. Here we pursue a similar approach to investigate the observability of the subdiurnal geocenter variations. Consider the geocenter variations in the inertial frame:

$$G = ge^{i\theta}$$

where $G = X_g + Y_g$ is the geocenter motion in the inertial frame, $g = x_g + iy_g$ is the geocenter motion in the body fixed frame, and θ is sidereal time. Then substituting the expressions from above for x_g in terms of prograde and retrograde terms and recognizing that:

$$\theta_s = \theta_s^* + m_s\theta$$

where m_s is 0, 1, or 2, for long period, diurnal, or semidiurnal tides, respectively, and θ_s^* is the tidal argument without the sidereal time term, yields:

$$G = \sum_{s=1}^{N}[g^{s+}e^{i\theta_s^*}e^{i[(m_s+1)\theta]} + g^{s-}e^{-i\theta_s^*}e^{-i[(m_s-1)\theta]}]$$

Therefore for $m_s = 1$, the second term will be long period in the inertial frame. This is the diurnal retrograde term, and as described above, it is possible for polar motion to express this term as a linear combination of orbit elements, and hence is unobservable. For geocenter, this term does not seem to be truly unobservable, but it is considerably weaker, as measured by the covariance of the adjusted values, even in simulation. The conclusion is that while one cannot express the geocenter variations exactly in terms of orbit elements, the orbit can partially accommodate such variations and the solution is weakened by the correlation.

Solution

A solution was performed as described above using nearly 18 years of Lageos data spanning the period May 1976 to February 1995. Over 700,000 three or two minute normal points from over 110 tracking sites formed this data set. The models used in the data analysis adhered generally to the latest IERS conventions [*McCarthy*, 1996], with several notable exceptions. The changes in the geopotential due to ocean tides were modelled using an enhanced model which included all terms, chosen from a degree and order 20 spherical harmonic expansion, with predicted perturbations that exceeded 1 mm in the radial, normal, or transverse components of the orbit. The nominal coefficients were derived from *Schwiderski* [1980], assuming linear admittances, with a subset of terms adjusted from a multisatellite solution. The solid tide model was expanded to include geopotential variations due to third degree terms assuming the Love number, k_3, of 0.093 [*Wahr*, 1981]. The free core nutation period was 430 days. The reference frame was fixed to the CSR95L01 system of *Eanes and Watkins* [1995] determined from analyses of the entire Lageos-1 and -2 missions. Site-dependent nuisance parameters such as range or clock biases were adjusted where necessary.

In the estimation process, mean orbital elements (excluding the nodal longitude), x_p, y_p, and UT1 were adjusted every 3 days, along a with single adjustment of the x and y prograde and retrograde geocenter variations and z geocenter variations for the nine tides described earlier. Earth orientation variations due to these same tides were also adjusted. In addition three diurnal tides and three semidiurnal tides with vanishingly small predicted geocenter signature due to their small amplitude were adjusted for use as controls in error analyses. These tides had Doodson numbers 166.554, 173.655, 153.655, 277.555, 263.655, and 225.855. With the above modelling and parametrization, the weighted post fit range residual rms over the entire data span was 23 millimeters, with fits in the later years generally around 15 millimeters.

Results

The results for the geocenter variations at each tidal frequency are presented in Table 1, along with their respective uncertainties. These uncertainties were derived by scaling the data weight assigned to the laser ranges so that the uncertainties for adjusted parameters for which external calibrations are available, such as site positions and velocities and subdaily variations in Earth orientation, are accurate. Therefore the uncertainties in Table 1 should be considered as reasonable best estimates rather than formal values.

Note the larger uncertainties associated with the retrograde diurnal geocenter variations, due to the difficulty in separating long period orbit error from these variations as discussed earlier. The observed variations for these terms, do not, however, appear unreasonably large. The variations in the z component are only slightly more poorly determined than their semidiurnal and prograde diurnal equatorial counterparts, due most likely to geometric distribution of the tracking network resulting in fewer high latitude sites.

A comparison of the observed geocenter variations (Table 1) with those predicted by the ocean tide models of *Schwiderski* [1980] (Sch), *Brosche and Wunsch* [1993] (BW), and with the ocean tide model CSR3.0 (CSR) recently computed from TOPEX/Poseidon altimeter is provided as Table 2. In this table, the values from Table 1, as well as the computed predictions from the ocean tide models have been converted to a consistent system to allow easy comparison. Finally, we note that the BW results were expressed in their paper with the opposite sign convention to the results in Table 1, and their convention has been adopted for Table 2. The conversions from Table 1 are expressed below.

$$g^{s+} = -A^{s+} e^{i(\phi^{s+} - \chi^s)}$$

$$g^{s-} = -A^{s-} e^{i(\phi^{s-} + \chi^s)}$$

$$z_g{}^s = A_z{}^s[\cos(\phi_z{}^s - \chi^s) + \sin(\phi_z{}^s - \chi^s)]$$

where the A's are the amplitudes and the ϕ's the phases given in Table 2 for the appropriate terms, and χ^s = +90 degrees for K_1, -90 for O_1, P_1, and Q_1, and 0 degrees for all semidiurnal tides.

The agreement between the Lageos observations and the ocean tide predictions, particularly those from the CSR3.0 TOPEX/Poseidon model is generally good. The agreement is a little worse for the retrograde diurnal terms. Finally, the Schwiderski prediction for M_2 is clearly an outlier with respect to the other tide models and the Lageos observations.

Error Analysis

Strong evidence in support of the above uncertainties is the small size of the estimates for the control tides. These tides should produce geocenter variations with amplitudes of less than 0.5 mm in all components. By assuming that all of the observed geocenter variations at these frequencies are due to other errors, we can obtain an estimate of their size. While the uncertainties were comparable to those obtained for the tidal frequencies presented in Table 1, the rms of the adjusted terms in the x and y components was 0.3 mm for the prograde diurnal variations, 0.5 mm for the retrograde diurnal variations, 0.2 mm for the prograde semidiurnal variations, and 0.2 mm for retrograde semidiurnal variations. These results give a good indication of the relative accuracy of the prograde and retrograde diurnal terms. The rms of the estimates for these tides in the z component was 0.3 mm for the diurnal terms and 0.2 mm for the semidiurnal terms. None of the adjusted variations for these tides exceeded twice their respective uncertainty.

Ocean loading errors are also a potential contributor to errors in the geocenter variations since they occur at the same tidal frequencies. However, due to the averaging of potentially random ocean loading errors at each site over the large Lageos tracking network, it may reasonably be assumed that the systematic effects would be small. To determine an upper bound on the effect of ocean loading errors, we have performed our solution with and without the entire IERS standard radial ocean loading model, and differenced the resulting geocenter variations. The rms differences were less than one millimeter for all components. We expect the effect of the horizontal terms to be even smaller.

Conclusions

The ability to observe tidally coherent variations in the position of the center of mass of the Earth with respect to the tracking site polyhedron located on the solid Earth at below the millimeter level has been demonstrated using several years of high quality tracking data for the Lageos satellite. The variations observed with most confidence generally agree well with those predicted by ocean tide models. The retrograde diurnal variations have been shown to be somewhat less accurate than the prograde terms, but not unobservable.

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Table 1. Observed Tidal Variations in the Geocenter

Tide	x_g^+	y_g^+	x_g^-	y_g^-	z_c	z_s
K_1	0.0 ± 0.4	-2.9 ± 0.4	-1.7 ± 1.2	-0.4 ± 1.2	5.2 ± 0.5	0.5 ± 0.5
P_1	-0.3 ± 0.4	0.7 ± 0.4	-0.6 ± 1.2	0.2 ± 1.2	-2.2 ± 0.5	-0.3 ± 0.5
S_1	0.4 ± 0.4	-0.2 ± 0.4	-2.0 ± 1.2	0.4 ± 1.2	-0.1 ± 0.5	0.0 ± 0.5
O_1	-0.5 ± 0.4	1.2 ± 0.4	1.1 ± 0.7	-1.6 ± 0.7	-3.3 ± 0.4	-0.5 ± 0.4
Q_1	-0.3 ± 0.4	0.1 ± 0.4	0.1 ± 0.5	-0.7 ± 0.5	-0.7 ± 0.4	0.0 ± 0.4
K_2	0.0 ± 0.4	-0.2 ± 0.4	-2.3 ± 0.7	0.4 ± 0.7	0.5 ± 0.4	0.6 ± 0.4
S_2	-0.5 ± 0.4	0.2 ± 0.4	-0.9 ± 0.7	-0.9 ± 0.7	-0.7 ± 0.4	-0.9 ± 0.4
M_2	-0.9 ± 0.4	0.6 ± 0.4	-1.5 ± 0.4	1.0 ± 0.4	-2.7 ± 0.4	-1.5 ± 0.4
N_2	-0.5 ± 0.3	0.3 ± 0.3	0.0 ± 0.4	0.7 ± 0.4	-0.4 ± 0.4	0.0 ± 0.4

All units are millimeters

Table 2. Comparison of Observed and Predicted Geocenter Variations

Solution		Prograde Amp	Prograde Pha	Retrograde Amp	Retrograde Pha	z Amp	z Pha
O_1	Sch	1.2	23	0.5	96	2.4	282
	BW	0.9	7	1.6	38	1.1	251
	T/P	1.0	21	0.6	80	2.9	278
	1,-1	1.2	24	2.0	35	3.3	278
P_1	Sch	0.6	11	0.2	79	1.3	286
	BW	0.6	9	0.4	56	1.4	287
	T/P	0.6	355	0.3	75	1.5	284
	1,-1	0.8	22	0.6	248	2.2	279
K_1	Sch	2.0	13	1.0	93	4.3	283
	BW	2.1	4	1.4	51	3.8	285
	T/P	1.9	353	1.0	81	4.6	286
	1,-1	2.9	0	1.8	105	5.2	276
N_2	Sch	0.7	323	0.5	250	0.3	10
	BW	0.3	12	0.2	238	0.8	28
	T/P	0.2	5	0.3	286	0.5	26
	1,-1	0.6	330	0.7	267	0.4	359
M_2	Sch	3.5	326	1.6	314	1.0	73
	BW	1.4	17	1.5	355	3.0	49
	T/P	0.8	355	1.4	300	2.1	45
	1,-1	1.1	323	1.8	326	3.1	29
S_2	Sch	1.6	270	0.4	49	0.4	52
	BW	0.7	320	0.9	352	1.3	130
	T/P	0.2	315	0.6	343	0.9	58
	1,-1	0.5	339	1.3	42	1.1	40

All amplitudes in millimeters, All phases in degrees